2016 Mathematics Standards Comments

As you read through this parent and teacher review of the draft 2016 Mathematics standards, please keep in mind the research of a well know developmental psychologist, Jean Piaget. Before he studied the minds of children, it was thought that children have the capability to think and reason like adults. However, after his studies on epistemology (the nature of knowledge) and how people come to gradually acquire it, he formed the Piaget theory –which is used by child psychologists around the world. Below is a basic chart explaining the cognitive development of children and what type of demands their minds can accept in their corresponding ages:

Preoperational		**2-7 yrs**	Once children acquire language, they are able to use symbols (such as words or pictures) to represent objects. Their thinking is still very egocentric though -- they assume that everyone else sees things from the same viewpoint as they do. They are able to understand concepts like counting, classifying according to similarity, and past-present-future but generally they are still focused primarily on the present and on the concrete, rather than the abstract.
Concrete Operational		**7-11 yrs**	At this stage, children are able to see things from different points of view and to imagine events that occur outside their own lives. Some organized, logical thought processes are now evident and they are able to: • order objects by size, color gradient, etc. • understand that if 3 + 4 = 7 then 7 - 4 = 3 • understand that a red square can belong to both the 'red' category and the 'square' category • understand that a short wide cup can hold the same amount of liquid as a tall thin cup However, thinking still tends to be tied to concrete reality
Formal Operational		**11+ yrs**	Around the onset of puberty, children are able to reason in much more abstract ways and to test hypotheses using systematic logic. There is a much greater focus on possibilities and on ideological issues.

A large number of the new 2016 Arizona Standards do not meet the criteria for clarity and appropriate cognitive demand. Specific problems in these areas will be pointed out below with the corresponding standard.

Also, parents, teachers and leading Mathematics Standards experts- Dr. James Milgram and Ze'ev Wurman- have voiced concerns that were missing from the 2010 Common Core standards that have been traditionally taught for decades and are necessary to assist children in future higher learning. The 2016 Mathematics Standards have continued this omission. These missing factors will also be listed in the appropriate grade in the specific standard review below.

Kindergarten

The criteria listed on the review of Kindergarten Math standards mentions: clarity, cognitive demand, and measurability. Developmental appropriateness was a concern many people nationwide and within AZ had about our previous standards. There is no mention of using developmental appropriateness as a criteria to evaluate the standards. There is only one reference to addressing a standard in relation to public comment, and no citations of research used despite the Executive Summary indicating these items would be used to review the standards. How can AZ parents be assured that with the Kindergarten standards looking largely the same in nature that the standards are developmentally appropriate for Kindergarten students?

In Counting/Cardinality (CC), there are few changes some being only for grammar/wording of the standard.

- **K.CC.A.3**- Dr. Milgram, "This is purely a reading standard, having nothing to do with Mathematics."
- **K.CC.B.4-** Dr. Milgram, "This is purely a vocabulary standard. Nothing wrong with it, just don't try to convince teachers that when they teach this, they are teaching 'mathematics.' "
- **K.CC.B.5** ...objects arranged in a line, a rectangular array, or a circle...This wording tells HOW to teach the standard not simply the WHAT to teach.
- **K.CC.B7**- Dr. Milgram, "Be more specific about what you mean by compare. If it is greater, less than or equal, etc." Wording needs to be modified.

In Operations and Algebraic Thinking, developmental appropriateness again was not mentioned as criteria used to evaluate a standard. There is no mention of public comment or research used. Most children cannot use "a variety of strategies" being that they are in the preoperational phase. They also cannot be expected to use equations to give answers to problems on their own. They need concrete ideas and lots of repetition.

- **K.OA.A.1**- tells HOW to teach not WHAT to teach even though one previous example was deleted
- **K.OA.A.2**- references using Table 1 which is full of HOW to teach not just WHAT to teach. Some examples were deleted under the heading, but the "variety of strategies" just moved to another place within the standards document-Table 1.
- **K.OA.A.2**- Ze'ev Wurman's comment, "Abstract equations are inappropriate for Kindergarten."
- **K.OA.A.3**- contains HOW to teach (drawings, objects) and that it must be done in multiple ways simply not WHAT to teach
- **K.OA.A.3**- Ze'ev Wurman's comment, "Abstract equations are inappropriate for Kindergarten."
- **K.OA.A.4**- contains HOW to teach (drawings, objects) and HOW to answer the question (drawings, objects) rather than simply the WHAT to teach.
- **K.OA.A.5-** is a good example of what 5 and 6 year old children can do. This specific line is also a good example of clarity. The rest of these "standards" are not really standards at all, they are prescribed methods of how to teach. It would be best to simply state what a child needs to know and learn, not HOW the teacher should teach and what method is to be used.

In Number and Operations Base Ten (NBT), few changes for grammar/wording, and some examples deleted. Only one mention of public comment but dismissed as being implied in the standard. No mention of evaluating standards for developmental appropriateness or research cited.

- **K.NBT.A.1**- tells HOW to teach not WHAT to teach. Parts of the example were deleted but HOW to remains with the inclusion of (sing objects, drawings)
- **K.NBT.B.2-** a new standard stipulates that adding and subtracting must be done using a variety of strategies. If a class understands conceptually using one strategy, the teacher is told that is not enough a HOW to says it must be done using many strategies.
- **K.NBT.B.2-** new standard needs to be reworded to not include "how to's"
- **K.NBT.B.3-** Overly prescriptive in telling a teacher how to teach the standard: "…count the number in each category and sort the categories by count."
- **K.NBT.B.4-** Again, overly prescriptive in methodology: "…using informal language to describe their similarities and differences."

Measurement and Data

- **K.MD.B.3-** It would be useful to know what the public was concerned with and how this standard addressed the public concern.

In Geometry (G), there were few changes and only a few examples deleted. No mention of developmental appropriateness or research cited.

- **K.G.A.2-** The word "correctly" seems unnecessary. The expectation is a student learn all the standards correctly not just this one.

The Standards for Mathematical Practice have ballooned in verbiage and are copied and pasted throughout K-12. In the lower grades, K-3 these goals are especially lofty and there is no citing of research or developmental appropriateness. These MP standards went from a simple vague phrase to a paragraph of HOW to's and imply that only proficient students can accomplish them by how it is worded. Standards are supposed to be the WHAT for all students. Teachers then differentiate the HOW to, but these paragraphs try to encompass and describe the EXAMPLES which the Public, and Standards Development Committee asked to be removed. These MP standards are not WHAT to teach, they are HOW, HOW MUCH TO DO, HOW A CHILD SHOULD THINK, RESPOND. **The MP Standards should be deleted in all grades.**

K.MP.1 through K.M.8- Kindergartners are not mathematically proficient students?! How does a Kindergartner know tools are relevant? Kindergartners cannot be expected to craft careful explanations! We don't want them to struggle and be frustrated at this level because they will just hate math from the start! Too much to ask of a Kindergartner. Standards for Mathematical Practice should not be the same for all grade levels. Delete for all grades!

These eight paragraphs have been presented to a large number of parents around Arizona. All have responded that these paragraphs are very difficult to understand and do not fit the criteria for clarity. "Reason abstractly" and "contextualize and decontextualize problems" are not directives to teach in concrete manners. Abstract thinking has no place in kindergarten. Also, the demand for kindergarteners to critique their peers and debate their reasoning is also inappropriate for their developmental stage. Kindergarteners are egocentric at this age and cannot see from another's perspective. Critiquing peers can lead to lowered self-esteem and loss of creativity. Debating with their peers is not cognitively appropriate at this age. And again, kindergarteners need concrete ideas presented to them and should not be required to reason and explain their answers.

The Standards for Mathematical Practice are also developmentally inappropriate for 1st grade, 2nd grade, and even 3rd grade when students are egocentric and whose minds have not developed past the pre-operational phase.

Missing: Patterns and Classification

- Establish concepts of likeness and difference by sorting and classifying objects according to various attributes: size, shape, color, amount, function, etc.
- Define a set by the common property of its elements.
- In a collection of objects that includes a given set and an item that does not belong, indicate which item does not belong.
- Moving from concrete objects to pictorial representations, recognize patterns and predict the extension of a pattern.
- Extend a sequence of ordered concrete objects.

The 2016 Math Standards draft for 1st grade, 2nd grade, and 3rd grade are all very similar to the Kindergarten Standards as far as not containing clear and understandable standards and having cognitive demand that is developmentally inappropriate causing undue stress on young children. The standards for all grades (K-12) are all overly prescriptive in telling a teacher how to teach rather than stating what the end goal in mind is. Again, this is where a standard no longer can be deemed as just a "standard" and instead becomes methodology.

First grade

The criteria listed on the review of First Grade Math standards mentions: clarity, cognitive demand, and measurability. Developmental appropriateness was a concern many people nationwide and within AZ had about our previous standards. There is no mention of using developmental appropriateness as a criteria to evaluate the standards. There is only one reference to adding a completely new standard (naming coins) in relation to public comment, and no citations of research used despite the Executive Summary indicating these items would be used to review the standards. Many examples were deleted but in a few places the Draft contains HOW to and Table 1 is filled with HOW to items. Many standards were just changed for wording, grammar, and even punctuation but the meaning and intent remained identical. There are many references of using multiple strategies which seem to emphasize knowing multiple ways is better than understanding the concept well using something salient to the student. It is often these strategies that get emphasized vs. learning the WHAT of the standard...the HOW of process takes over. The Mathematical Practice Standards copied and pasted in each grade are especially questionable in lower grades K-3 and should be omitted in all grades. The MP standards are filled with strategies and HOW to's, HOW to think, HOW a student should respond. How can AZ parents be assured that with the First Grade standards looking largely the same in nature that the standards are developmentally appropriate for First Grade students?

In Operations and Algebraic Thinking (OA), developmental appropriateness again was not mentioned as criteria used to evaluate a standard. Also no mention of public comment or research used.

- **1.OA.C.5-** relate counting to addition and subtraction by counting on 2 to add 2: good to see this deleted as it was a strategy not a standard.
- **1.OA.C.5-** the new standard is clear Fluently add and subtract through 10 w/all examples deleted
- **1.OA.A.1-** tells HOW to teach not WHAT to teach even though previous examples were deleted, they moved to Table 1 and mention using a variety of strategies.
- **1.OA.A.1**- Dr. James Milgram and Ze'ev Wurman stated, "teaching this standard alone should consume perhaps 80% of time in the first grade!" This is a standard within a standard and very unclear as written.
- **1.OA.A.2-** references using Table 1 which is full of HOW to teach not just WHAT to teach. Some examples were deleted under the heading, but the "variety of strategies" just moved to another place within the standards document-Table 1
- **1.OA.B.4-** contains HOW to teach with reference to Table 1. Examples of HOW to strategies moved locations.

In Number and Operations Base Ten (NBT), few changes for grammar/wording, and some examples deleted. No mention of public comment. No mention of evaluating standards for developmental appropriateness or research cited. Many parents cited concerns with how NBT was demonstrated in curriculum (which is not the scope of this review) but a concern is by keeping this standard coded exactly the same, a teacher can use strategies that parents didn't like and felt were developmentally inappropriate or cumbersome...ie make a ten strategy, friendly numbers ending in zero with lots of grouping of numbers to do simple addition/subtraction problems.

- **1.NBT.C.4-** tells HOW to teach not WHAT to teach. It mentions using models and/or strategies which implies a teacher must introduce multiple methods, but it may be that students understand the concept using one strategy well.
- **1.NBT.C.4-** Dr. Milgram, "Probably this indicated that one should change the bound of the key standard in line 11 to 100 from 20. Then the rest of this standard becomes four substandards."
- **1.NBT.C.5-** tells HOW to teach not WHAT to teach. It requires a student to explain his thinking. Having been in grade 1 classrooms, many kids cannot do this using a strategy using a number line or a 100's chart, but can with counting 10 blocks. This is developmentally questionable as a universally achievable skill in grade 1. Students may be able to via rote memory recite 10, 20 30 etc but explaining it is very abstract at age 6/7.
- **1.NBT.C.6-** is HOW to teach not WHAT to teach. Again saying models and multiple strategies to be used. If a class understands conceptually using one strategy, the teacher is told that is not enough a HOW to says it must be done using many strategies.
- **1.NBT.C.7-** is HOW to not WHAT to teach. Again saying models and multiple strategies to be used.
- **1.NBT.C.7**- New standard- Too prescriptive on "how to teach" not "what to teach." Modify language.
- **1.NBT.MD.A.2-** tells HOW to measure by saying laying multiple copies of a shorter object end to end. Using a ruler seems to be discouraged by the description of HOW to teach this standard.
- **1.MD.B.4-** good standard addition identifying coins by name and value per public request.

In Geometry (G), there were few changes and only a few examples deleted. No mention of developmental appropriateness or research cited.

- **1.G.A.1-** Dr. Milgram. "Probably too much for first grade. If you think otherwise, then you should indicate the research that supports this standard."

The Standards for Mathematical Practice have ballooned in verbiage and are copied and pasted throughout K-12. In the lower grades, K-3 these goals are especially lofty and there is no citing of research or developmental appropriateness. These MP standards went from a simple vague phrase to a paragraph of HOW to's and imply that only proficient students can accomplish them by how it is worded. Standards are supposed to be the WHAT for all students. Teachers then differentiate the HOW to, but these paragraphs try to encompass and describe the EXAMPLES which the Public, and Standards Development Committee asked to be removed. These MP standards are not WHAT to teach, they are HOW, HOW MUCH TO DO, HOW A CHILD SHOULD THINK, RESPOND. **The MP Standards should be deleted in all grades.**

2nd grade

Generally the standard descriptions remain relatively the same. Some clarifying language has been added and/or prescriptive language or "how to's" have been removed. No research was cited or appeared to be used throughout 3rd grade standards

Operations and Algebraic Thinking

- **2.OA.A.1-** Dr. James Milgram and Ze'ec Wurman stated, "teaching this standard alone should consume perhaps 80% of time in the 2nd grade!" This is a standard within a standard and very unclear as written.
- **2.OA.B.2-** Where is 1.OA.6- I could not find?? Not developmentally appropriate to use "mental strategies" in 2nd grade. Where is research to back this up? Will frustrate and confuse a 2nd grade student!

- **2.OA.C.3-** Dr. Milgram, "I don't know what the last sentence here is trying to do except confuse." Reject or rewrite?

Measurement and Data

- **2.MD.A.2-** Dr. Milgram, "Might be pretty sophisticated in second grade. Show the research that demonstrates that this is appropriate, or move up to third or even fourth grade."
- **2.MD.D10**- Table 1 remains which is prescriptive or "how to's" not "what to teach."

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3rd grade

Generally the standard descriptions remain relatively the same. Some clarifying language has been added and/or prescriptive language or "how to's" have been removed. No research was cited or appeared to be used throughout 3rd grade standards

Operations and Algebraic Thinking

- **3.OA.D.9-** Is this developmentally appropriate to "assess reasonableness and estimation strategies in 3rd grade?" Show research to back this up!

Measurement and Data

- **3.MD.A.1-** Where are group notes? It appears example was just removed?
- **3.MD.3.7-** Third grade students do not understand "real world" problems nor "mathematical reasoning." Not developmentally appropriate for a 3rd grader! Where is research to back this up?
- **3.MD.D.8 (renamed 3.MD.C.9)-** Third grade students do not understand "real world" problems nor "mathematical reasoning." Not developmentally appropriate for a 3rd grader! Where is research to back this up?

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4th grade

Generally the standard descriptions remain relatively the same. Some clarifying language has been added and/or prescriptive language or "how to's" have been removed. Some research evidence has been included from EDThoughts What We Know About Education and Learning edited by the Mid-continent Research for Education and Learning (McREL): "An algorithm is a precise, step-by-step method or set of rules for solving problems of a particular type...there are algorithms of many types."(Pg. 82).

Operations and Algebraic Thinking

- **4.OA.A.2-** Standard still contains prescriptive examples or "how to's" with Table 2 included.
- **4.OA.A.3-** Is this standard developmentally appropriate and research-based to use "mental computation" and to think algebraically for a fourth grader?
- **4.OA.B4-** This is a standard within a standard. Can it be separated out or broken down into several standards?

Numbers and Operations in Fractions

- **4.NF.C.7-** Added use number sense of decimal fractions to assess the reasonableness of answers to create consistency between **NF.A.2 and NF.C.7**. How is the reasonableness of answers determined?

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5th grade

Generally the standard descriptions remain relatively the same. Some clarifying language has been added and/or prescriptive language or "how to's" have been removed. Some research evidence has been included from EDThoughts What We Know About Education and Learning edited by the Mid-continent Research for Education and Learning (McREL): "An algorithm is a precise, step-by-step method or set of rules for solving problems of a particular type...there are algorithms of many types." (Pg. 82); and on page 14, "Representations of mathematical ideas can be visual, including equations, graphs, pictures and charts...students with well developed understanding of a concept can represent it in a variety of ways (Page 14)."

Number and Operations in Base 10

- **5.NBT.B6**- Standard description is relatively the same. Prescriptive language removed. What supporting document is referred to here?
- **5.NBT.B7**- Prescriptive language or "how to's" still in the standard calling our models and drawings to be used instead of just "what to teach."

Numbers and Operations- Fractions

- **5.NF.A.2**-Use benchmark fractions and number sense fractions to estimate mentally and assess the reasonableness of answers. How is a student going to be measured on "assessing the reasonable of their answers?"

Measurement and Data

5.MD.C5- Does a 5th grader understand "real-world problem solving and how to link that to everyday work and decision making?" Where is the research to back this up?

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Missing Kindergarten - Grade 7 (Dr. James Milgram):

- CC does not require proficiency with addition and subtraction until Grade 4 (a grade behind our international competitors).
- CC does not require proficiency with multiplication using the standard algorithm (step-by-step procedure for calculations) until Grade 5 (a grade behind standard expectations).
- CC does not require proficiency with division using the standard algorithm until Grade 6 (two grades behind our international competitors).
- CC starts teaching decimals in Grade 4 (about two years behind the more rigorous states).
- CC fails to teach in K-7 key geometrical concepts (e.g., sum of angles in a triangle, isosceles and equilateral triangles, etc.).
- Excludes fluent conversion between different forms of fractions – regular fractions, decimals, and percents.
- CC fails to teach prime factorization. Consequently, it does not include teaching about least common denominators or greatest common factors.
- Compound interest and the associated formula, (x^(n+l) - l)/(x-l) = 1 + x + x^2 + ... + x ^n. This is or used to be a seventh grade or at latest, eighth grade topic.

Algebra I, Algebra II and High School Plus

General Items

- The goal is to have the standards be easily understood by faculty, students, and parents. Many of the standards would be difficult for parents to understand (unless they completed upper division math courses in college). Examples would go a long way in helping all to best understand what the expectations are. Currently few examples are provided.

- It would be beneficial to cover less material but to apply that material to real life situations. Applying concept to real life requires students to learn material on a deeper level and allows them to see the usefulness of mathematics outside of the classroom.
- From **HS.A-REI.A.1** example. (explain each step). Removing questions such as "What is the fifth step in the process?" Some students solve equations or complete processes differently. We should not limit students to only one method of completing a math problem.
- At times it is ambiguous what class should cover certain items. For HS.F-IF.B.4 that concept is covered in Algebra 1, Algebra 2 and upper division math. We need a breakdown of how far Algebra 1 needs to go, what Algebra 2 needs to cover, etc.

Specific Items

- **HS.A.SSE.B.3-** a Completing the square should be moved to Algebra 2. This unit would fit nicely into solving/factoring unit.
- **HS.F-IF.A.1-** Domain and rage should be introduced in Algebra 1 but not in depth. In depth domain and range should be moved to Algebra 2.
- **HS.F-IF.A.2-** Function notation should be moved to Algebra 2. Even an introduction would not benefit students much. All of it should be moved to Algebra 2.
- **HS.F-IF.C.7b** Graphing piecewise functions, square root functions, and cube root functions should all be moved to Algebra 2. Algebra 1 should cover linear and quadratic graphs.
- **HS.F-TF.A.1, HS.F-TF.A.2, and HS.F-TF.B.5** should be moved from Algebra 2 to PreCalculus. The unit circle and radians fits into material in Precalculus/trigonometry. It would not fit into Algebra 2 material.
- **HS.F-TF.C.8-** should be moved to Precalculus. Trigonometry items are covered as part of the Precalculus course.
- **HS.S-IC.B.4 AND HS.S-IC.B.5-** should be deleted from the required curriculum. We have too many items to cover in a given year. Deleting these items would allow us to focus on deeper learning of other material throughout the year.
- **HS.S-IC.B.6-** This section should be moved to Algebra 1. It fits perfectly with mean, median, and mode.

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Missing Algebra 1: Missing components needed for Algebra 2 and Calculus: (Dr. James Milgram):

1. Division of monomials and polynomials (only addition/subtraction/multiplication are covered)
2. Derivation and understanding of slopes of parallel and perpendicular lines
3. Manipulation and simplification of rational expressions
4. Multi-step problems with linear equations and inequalities

5. Multi-step problems with four operations between polynomials
6. Multi-step problems involving manipulation of rational expressions
7. Solving two linear inequalities in two variables and sketching the solution sets

The following were added to California's Common Core version:

1. Solve problems with equations and inequalities with absolute value
2. Solve problems with quadratic expressions

Missing Algebra 2: Some key topics missing (Dr. James Milgram):

1. Writing quadratic polynomials in two or three variables as sums or differences of perfect squares. (KEY for the study of conic sections, which, in turn, underlies almost everything that is done in STEM areas.)
2. Detailed study of surfaces of revolution coming from quadratic polynomials as described above. In particular, the focus here should be on parabolic mirrors and their applications.
3. Introduction of the foci and the directorix for conies and their applications to parabolas and parabolic mirrors, and also for ellipses and elliptic surfaces with applications to things like whispering galleries and Kepler's laws.
4. Definition and implications of the eccentricity for conic sections.
5. Structure of logarithms to base 10, e, or general base, b. Conversion between bases, calculation of explicit values in simple cases.

Missing Pre-calculus and/or Algebra 2 and trigonometry (Dr. James Milgram):

1. Partial fraction decomposition of relatively simple rational functions and their graphs. Specifically, Understand that a function of the form (ax + b)/((x-r)(x-s)) can always be written as a sum (l/(x-r)) + (m/(x-s)), where, in this case 1 + m = a, and rm + ls = -b. Apply this to the determination of the graphs of such functions.
2. Graph functions in polar coordinates. Key examples, circles (r = 2cos(t)), Cardioids (2+ 2cos(t) = r), Rose petal curves (r = sin5t), lemniscate ($r^2 = 4\sin(2t)$).

Missing Algebra 2: Missing components needed for Calculus (Dr. James Milgram):

- composite functions
- combinations and permutations
- finite and infinite arithmetic and geometric sequences
- mathematical induction

Note that all four topics above are quite "formal" in line with the overly formal treatment of algebra in Core Standards. The topics sketched in above are much more "realistic" in terms of the actual needs of students wishing to major in ANY technical area in college.

See: http://concernedpvparents.org/2014/05/27/cc-math-dumbed-down-proof/

Geometry

I have both a Bachelor of Science and Master of Arts degree in Secondary Mathematics Education and have taught high school and college mathematics for 38 years with more than 20 years of that in geometry (Tucson teacher). I have taught geometry from both the Euclidean approach and the Transformational approach. Blending the two is appropriate and beneficial but a balance is necessary. **The geometry curriculum is significantly broad and by necessity needs to be trimmed to fit within the time constraints of the school year**. The old statement that a course is a mile wide and an inch deep was never more obvious than in geometry. To gain the depth we must shorten the width. **Some concepts might be moved down to lower grades but much should be moved to upper grades. In addition, the Arizona standards need to be clear,**

concise and understandable to all the stake holders: politicians, administrators at both the local and state level, teachers, parents, and students. After all it takes a village to educate a child. I have spent a great deal of time reading and rewriting the standards so that I understood what was expected, imagine a politician or a parent trying to understand what is stressed.

Congruence (G-CO)

G.G-CO.A Experiment with transformations in the plane.

The Arizona College and Career Ready Standards describe high school geometry as primarily Euclidean, yet there appears to be a significant reliance on transformations.

- **G.G-CO.A.2** -Although transformations are important they are not the main focus of Euclidean geometry, but instead are a visual and special method of understanding the theorems and postulates of geometry. I believe Arizona's standards are over emphasizing transformations to the exclusion of virtually other approach.

Overview (reworked for clarity and brevity)

- **G.G-CO.A.1 -** Geometry emphasizes an understanding of the attributes and relationships of geometric objects which can be applied in diverse contexts – interpreting a technical drawing, estimating the amount of wood needed to frame a house, or drawing computer graphics. There are many types of geometry but school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates).
- **G.G-CO.A.4-** Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes, as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

G.G-CO.B Understand congruence in terms of rigid motions.

- **G.G-CO.B.6-** During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental to this study are the rigid motions: translations, rotations, reflections, and combinations thereof. All are assumed to preserve distance and angle measure (and therefore shapes).
- **G.G-CO.B.7 -** Two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. Once the triangle congruence criteria (SSS, SAS, ASA, and AAS) are established, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. In advanced classes the SSA congruence criterion can be studied. Although it is not a universal congruence criterion it does create congruence under specific conditions and is the basis for the ambiguous case of the Law of Sines.
- **G.G-CO.A.5 -** Similarity transformations (rigid motions followed by dilations) define similarity, formalizing it as "same shape" and "scale factor". These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

Similarity, Right Triangles, and Trigonometry (G-SRT)

- **G.G-SRT.B.4-** The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions.
- **G.G-SRT.A.3-** This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra.

Expressing Geometric Properties with Equations (G-GPE)

- **G.G-GPE.B.4 -** Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

The Standards for Mathematical Practice have ballooned in verbiage and are copied and pasted throughout K-12. In the lower grades, K-3 these goals are especially lofty and there is no citing of research or developmental appropriateness. These MP standards went from a simple vague phrase to a paragraph of HOW to's and imply that only proficient students can accomplish them by how it is worded. Standards are supposed to be the WHAT for all students. Teachers then differentiate the HOW to, but these paragraphs try to encompass and describe the EXAMPLES which the Public, and Standards Development Committee asked to be removed. These MP standards are not WHAT to teach, they are HOW, HOW MUCH TO DO, HOW A CHILD SHOULD THINK, RESPOND. **The MP Standards should be deleted in all grades.**

Missing Geometry: Some key topics missing (Properties of triangles and circles) [Dr. James Milgram]:

1. Students should know that every triangle is circumscribed by a unique circle with center at the intersection point of the three perpendicular bisectors of the edges (also, that all three DO intersect in a single point).
2. They should know that every right triangle has the center of the circumscribing circle on its hypotenuse, and conversely.
3. They should know that the angle subtended by an arc on the circle (the angle obtained by drawing the two lines from the center to the ends of the arc), is twice the angle subtended by the ends of the arc and any point in the complement of the arc.

General Overview Comments

It is evident, after reviewing the Draft 2016 AZ ELA and Math Standards, that the review committees are hard-working and committed individuals who are worthy of this trust to revise and edit the 2010 Standards. Below you will find five main responses of how the standards need to advance for the health, benefit and success of our students.

1. Developmental Expert Consultation: Originally with Common Core, there were not enough experts in education or child development on the board of its creation. Inclusion of child development experts, those who understand developmental stages and the variances of learning that take place within each grade level, is vital to the progression of the new standards. Specialists of this kind would guide our state

against overloading our children to the point of burn out and anxieties that could be harmful. The 2016 standards revision workgroups did incorporate "numerous models and sources, including state departments of education, scholars, K-12 teachers, academic and instructional coaches, curriculum directors, administrators, university professors, parents, students, and other members of public" (2016 ELA Draft Introduction 1), however, upon adding the new written standards in the 2015 ELA Draft and retaining primarily the same structure as the 2010 Common Core Standards. Have developmental experts played a significant part in this process?

2. Parent Involvement and Notification Guaranteed: Parents have been involved in the revision and feedback of the new 2016 ELA and Math Standards, however, in the future is parent involvement and incorporation guaranteed? This is not seen in the new standards. Parental concern of what resources and texts should be used, as well as, how the curriculum should be taught could easily and conveniently be overlooked by the schools and districts, leaving the parent unable to influence what they find to be unacceptable or inappropriate for their child. The curriculum, as well as the state level standards, may also shift and adjust as time evolves. How will parents know their voice will be heard and incorporated into any future revisions? Will they be given adequate notification of upcoming changes or revisions?

3. Educational Vision More than College/Career Readiness: While the standards and curriculum, as presented in the Drafted 2016 ELA and Math Standards, are designed to "demand high levels of reason and thinking" (2016 ELA Draft Executive Summary 1) and prepare them to "succeed in credit-bearing, college-entry courses and/or in the workplace" (2016 ELA Draft Introduction 1) how will we be sure that other programs and subject areas important to student growth and development won't get pushed to the side or cut. Is there more to education? Education translates into a Latin base "educare" which means "to bring up," "a rearing," might this mean more than college and career readiness? What about being a responsible citizen or developing important life skills--such as budgeting and paying taxes?

4. High Stakes Testing Needs to be Strictly Limited (or Deleted): Is all this testing really necessary and helpful for student learning? According to a study titled "High-Stakes Testing and Student Achievement: Does Accountability Pressure Increase Student Learning?" conducted by researchers at the University of Texas at San Antonio and Arizona State University, it claimed "no consistent evidence that high-stakes testing works to increase achievement" (Nichols, Class, G.V, & Berliner, D.C. 2006). From my own experience as a teacher and parent of children enrolled in Arizona schools, high stakes testing impedes on genuine learning and teaching. It places a pressure on students and teachers that is primarily task based and not inquiry based, thus making internal learning artificial at best.

5. PRIVACY of OUR CHILDREN and their FAMILIES: If we do not know what is on a statewide test and/or survey and are not sure we can, with full confidence, protect the opinions, beliefs, descriptions and personal history and/or information of our children and their families then the test should not be administered into AZ schools. This is a trust that our schools, teachers, students and families desperately need. This cannot be emphasized enough.

Citations:

Nichols, S. L., Glass, G. V, & Berliner, D. C. (2006). High-stakes testing and student achievement: Does accountability pressure increase student learning? Education Policy Analysis Archives, 14(1). Retrieved [date] from http://epaa.asu.edu/epaa/v14n1/.